

$$\sin^6 x + \cos^6 x = \sin x \cos x$$

$$\cos^2 x = (\cos 2x + 1)/2$$

$$\sin^2 x = (1 - \cos 2x)/2$$

$$(\sin^2 x)^3 + (\cos^2 x)^3 = \sin x \cos x$$

$$((1 - \cos 2x)/2)^3 + ((\cos 2x + 1)/2)^3 = \sin x \cos x$$

$$(1 - \cos 2x)^3 / 8 + (\cos 2x + 1)^3 / 8 = \sin x \cos x \quad | \cdot 8$$

$$(1 - \cos 2x)^3 + (\cos 2x + 1)^3 = 8 \sin x \cos x$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$((1 - \cos 2x) + (\cos 2x + 1))((1 - \cos 2x)^2 - (1 - \cos 2x)(1 + \cos 2x) + (\cos 2x + 1)^2)$$

$$(1 - \cos 2x + \cos 2x + 1)((1 - 2\cos 2x + \cos^2 2x) - (1 + \cos 2x - \cos 2x - \cos^2 2x) + (\cos^2 2x + 2\cos 2x + 1))$$

$$2(1 - 2\cos 2x + \cos^2 2x - 1 - \cos 2x + \cos 2x + \cos^2 2x + \cos^2 2x + 2\cos 2x + 1)$$

$$2(1 + 3\cos^2 2x) = 8 \sin x \cos x \quad | :2$$

$$1 + 3\cos^2 2x = 4 \sin x \cos x$$

$$1 + 3(1 - \sin^2 2x) = 2 \sin 2x$$

$$1 + 3 - 3\sin^2 2x = 2 \sin 2x$$

$$3\sin^2 2x + 2\sin 2x - 4 = 0$$

$$\sin 2x = t$$

$$3t^2 + 2t - 4 = 0$$

$$D = 1 + 12 = 13$$

$$t_1 = (-1 + \sqrt{13})/3$$

$$t_2 = (-1 - \sqrt{13})/3 \quad \text{не подходит}$$

$$\sin 2x = (-1 + \sqrt{13})/3$$

$$x = (\arcsin(-1 + \sqrt{13})/3)/2 + Pn$$

$$x = P/2 - (\arcsin(-1 + \sqrt{13})/3)/2 + Pn$$

Ответ: $(\arcsin(-1 + \sqrt{13})/3)/2 + Pn$; $P/2 - (\arcsin(-1 + \sqrt{13})/3)/2 + Pn$

$$\sin^5 x + \cos^5 x = 1$$

$$\sin^5 x + \cos^5 x = \sin^2 x + \cos^2 x$$

$$\sin^5 x - \sin^2 x + \cos^5 x - \cos^2 x = 0$$

$$\sin^2 x(\sin^3 x - 1) + \cos^2 x(\cos^3 x - 1) = 0$$

$$\sin^2 x((\sin x - 1)(\sin^2 x + \sin x + 1)) + \cos^2 x((\cos x - 1)(\cos^2 x + \cos x + 1)) = 0$$

$$(1 - \cos^2 x)((\sin x - 1)(\sin^2 x + \sin x + 1)) + (1 - \sin^2 x)((\cos x - 1)(\cos^2 x + \cos x + 1)) = 0$$

$$(1 - \cos x)(1 + \cos x)(\sin x - 1)(\sin^2 x + \sin x + 1) + (1 - \sin x)(1 + \sin x)(\cos x - 1)(\cos^2 x + \cos x + 1) = 0$$

$$(-1)(\cos x - 1)(1 + \cos x)(\sin x - 1)(\sin^2 x + \sin x + 1) + (-1)(\sin x - 1)(1 + \sin x)(\cos x - 1)(\cos^2 x + \cos x + 1) = 0$$

$$(\cos x - 1)(1 + \cos x)(\sin x - 1)(\sin^2 x + \sin x + 1) + (\sin x - 1)(1 + \sin x)(\cos x - 1)(\cos^2 x + \cos x + 1) = 0$$

$$(\sin x - 1)(\cos x - 1)((1 + \cos x)(\sin^2 x + \sin x + 1) + (\sin x + 1)(\cos^2 x + \cos x + 1)) = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

$$x = P/2 + 2Pn$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$x = 2Pn$$

$$(1 + \cos x)(\sin^2 x + \sin x + 1) + (\sin x + 1)(\cos^2 x + \cos x + 1) = 0$$

$$1 + \cos x \geq 0$$

$$\sin x + 1 \geq 0$$

$$\sin^2 x + \sin x + 1 > 0$$

$$\cos^2 x + \cos x + 1 > 0$$

$$1 + \cos x = 0$$

$$\sin x + 1 = 0$$

Решения нет, так как \sin и \cos одного угла не могут одновременно быть -1

Ответ: $P/2 + 2Pn$; $2Pn$